Unit 3.1

Introduction to combinational circuits.
Boolean Algebra

- Boolean Algebra
- Logical Operations:
  OR, AND, XOR and NOT
- Boolean Algebra
  - Postulates
  - Theorems
- Logical functions. Canonical forms.
- Truth tables
- Universality of NAND and NOR gates

Boolean Algebra. Definition

- **Boolean Algebra** (after G. Boole 1779-1848)**:**
  Algebra in which:
  - the **logical variables** can take just two values: (**0** and **1**).
  - Three **logical operations** are defined:
    - Logical negation, **NOT**
    - Logical addition. **OR**, +
    - Logical multiplication, **AND**. *
- It is the algebra that computers use

Logical operations

- **OR**

a	b	a OR b
0	0	0
0	1	1
1	0	1
1	1	1

- **NOT**

a	NOT a
0	1
1	0

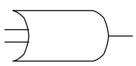
- **AND**

a	b	a AND b
0	0	0
0	1	0
1	0	0
1	1	1

- **XOR**

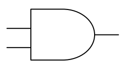
a	b	a XOR b
0	0	0
0	1	1
1	0	1
1	1	0

Logical gates (I)



a	b	a OR b
0	0	0
0	1	1
1	0	1
1	1	1

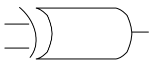
Represented as: **a + b**



a	b	a AND b
0	0	0
0	1	0
1	0	0
1	1	1

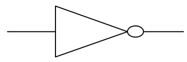
Represented as: **a · b**

Logical gates(II)



a	b	a XOR b
0	0	0
0	1	1
1	0	1
1	1	0

Represented as: $a \oplus b$



a	NOT a
0	1
1	0

Represented as : $\overline{a}$

Boolean Algebra Postulates

1. Commutative law.
   - **a + b = b + a** **a * b = b * a**
- 2. Distributive law
   - **a*(b+c) = a*b + a*c**
   - **a+(b*c) = (a+b) * (a+c)**
- 3. Identity element
   - **a + 0 = a** **a * 1 = a**
- 4. Complement
   - $a + \overline{a} = 1$ $a * \overline{a} = 0$

Boolean Algebra Theorems

The following theorems can be proved out of the previous postulates

- 1. Idempotence:
  - $a+a = a$ $a*a = a$
- 2. Commutativity:
  - $a + \overline{b} = b + a$ $a * \overline{b} = b * a$
- 3. Associativity:
  - $a+(b+c) = (a+)b+c$ $a*(b*c) = (a*b)*c$
- 4. Distributivity:
  - $a*(b+c) = a*b + a*c$ $a+(b*c) = (a+b) * (a+c)$

Boolean Algebra Theorems

- 6. Absorption:
  - $(a*b)+a = a$ $(a+b)*a = a$
- 8. Annihilator:
  - $a+1 = 1$ $a*0 = 0$
- 9. Double negation
  - $\overline{\overline{a}} = a$
- 10. De Morgan's laws:
  - $\overline{a+b} = \overline{a} * \overline{b}$ $\overline{a*b} = \overline{a} + \overline{b}$

  They make possible to shift from logical sums to logical products and viceversa

Boolean Algebra
Logical functions. Canonical forms

- **Logical functions:** mathematical expression in terms of Boolean variables related by logical operations. Example**:**

  $f_1(c,b,a) = a + c \cdot b + c \cdot b \cdot a$

- **Canonical term:** term in the function (product or sum) in which all variables (or their complement) appear.
- **Canonical Function:** Function in which all terms are canonical
- **Minterm (Sum of Products, POS):** canonical term in form of sum of products of variables (ej.: c·b·a).
  - **Conversion:** Multiply each non canonical term by the sum of missing variables (both natural and complemented).
- **Maxterm (Product of Sum, POS):** canonical term in form of product of sums of variables (ej.: c+b+a).
  - **Conversion:** Add to each non canonical term a product formed by the missing variables (both natural and complemented).

Truth table

- Way to represent a logical function in which the value of all combinations of variables appears. The canonical form can be easily obtained from it.

  Example:

$f(c,b,a) = c \cdot b + \bar{c} \cdot a$

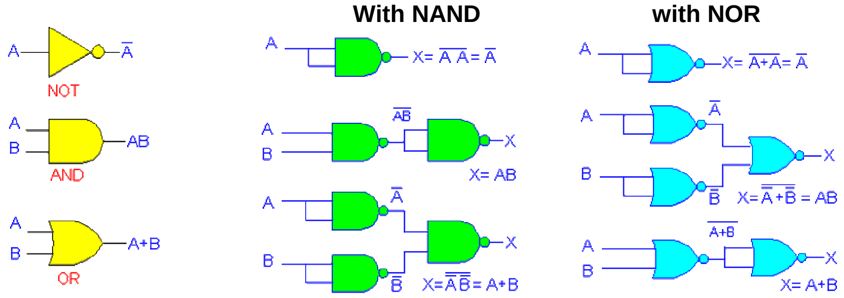
Truth table:

	c	b	a	f
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

Universality of NAND and NOR (I)

- NAND y NOR gates are **universal**: any logical function can be expressed just with NAND gates or just with NOR gates.

With NAND **with NOR**

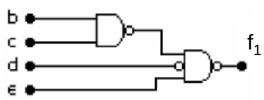


Universality of NAND and NOR (II)

- To build any circuit with NAND or NOR gates, double negation and De Morgan laws are used.
- Example: $f_1 = bc + d + \bar{e}$
- With NAND

  Switch to sums of products

  $f_1 = \overline{\overline{bc+d+\bar{e}}} = \overline{\overline{bc}\cdot\bar{d}\cdot\bar{\bar{e}}} = \overline{\overline{bc}\cdot\bar{d}\cdot e}$



- with NOR

  Switch to products of sums

  $f_1 = \bar{\bar{bc}}+d+\bar{e} = \overline{\bar{b}+\bar{c}}+d+\bar{e} = \overline{\overline{\overline{\bar{b}+\bar{c}}+d+\bar{e}}}$