

Unit 2: Binary Numbering Systems

- Definitions
- Number bases
- Numerical representations. Integer fixed point.
 - Binary
 - 2's complement
 - BCD
 - Addition-subtraction
- Alphanumerical representations

Fundamentals of Computer Technology

Basic Bibliography

Any book on digital electronics, for instance:

- Digital Fundamentals. (Chapter 2)
T.L. Floyd
Pearsons Prentice Hall



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Definitions

- **Space of a representation:** number of bits to store a data (numerical or character)
 - **Byte** (8 bits)
 - **Word** (n bits, generally 16, 32, 64)
- **Range of representation:** Maximum and minimum value that can be represented in a numbering system with fixed number of digits
- **Resolution of the representation:** Difference between a number and the next one in the representation
- **Code length:** number of elements that can be represented with a n -bit representation (example: for pure binary with n bits the code length is 2^n)



Numbering bases (I)

- Bases 2, 8, 10 y 16

Binary (base 2)	Octal (base 8)	Decimal (base 10)	Hexadecimal (base 16)
0	0 (000)	0 (0000)	0 (0000) A (1010)
1	1 (001)	1 (0001)	1 (0001) B (1011)
	2 (010)	2 (0010)	2 (0010) C (1100)
	3 (011)	3 (0011)	3 (0011) D (1101)
	4 (100)	4 (0100)	4 (0100) E (1110)
	5 (101)	5 (0101)	5 (0101) F (1111)
	6 (110)	6 (0110)	6 (0110)
	7 (111)	7 (0111)	7 (0111)
		8 (1000)	8 (1000)
		9 (1001)	9 (1001)



Numbering bases (II)

P ₇	P ₆	P ₅	P ₄	P ₃	P ₂	P ₁	P ₀
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The position of each bit represent its weight



Units: 10^0

Tens: 10^1

Hundreds: 10^2

Thousands: 10^3

Tens of Thousands: 10^4

To compute the decimal value:

$$value = \sum_{i=0}^{n-1} x_i \cdot base^i$$

▪ Examples:

▪ Binary number 10101.

Decimal value:

$$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 21$$

▪ Hexadecimal number :78A

Decimal value:

$$7 \times 6^2 + 8 \times 16^1 + 10 \times 16^0 = 1930$$



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Fixed point representations Pure Binary

n=8 bits							
X ₇	X ₆	X ₅	X ₄	X ₃	X ₂	X ₁	X ₀
n-1							0

▪ Base 2 **positional system** for integers

▪ **Weights** are: $P_i = 2^i$

▪ **Decimal value** with n bits: $Value = \sum_{i=0}^{n-1} 2^i \cdot x_i$

▪ **Range:** $[0, 2^n - 1]$

▪ **Resolution** = 1



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Fixed point representations 2's Complement (2C)

- **Positive numbers:** start with 0, represented in pure binary
- **Negative numbers:** start with 1, represented in 2C
- Then the **MSB (Most Significant Bit)** indicates the **sign**, but for operations all n -bits are treated alike
- To **represent a negative number**: **- A = 2C of A**. Operations to obtain 2C:
 - Obtain **1C** (1's complement) of A : $\bar{A} = 2^n - A$ (equivalent to replace $0 \leftrightarrow 1$)
 - Add 1: $\bar{A} + 1$
- To obtain the **decimal value** (n bits):
$$Value = \begin{cases} + \sum_{i=0}^{n-1} 2^i \cdot x_i & \text{if } x_{n-1} = 0 \\ -Value(2C(number)) & \text{if } x_{n-1} = 1 \end{cases}$$
- **Range:** $[-2^{n-1}, -1, 0, (2^{n-1} - 1)]$
- **Resolution** = 1



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Addition-Subtraction in 2's complement

- Main reason to use 2C is that **addition and subtraction operations are simplified**:
 - Operate without taking into account the sign of the operands
 - Final carry is ignored.
 - To subtract just add the 2C of the number: $A - B = A + 2C(B)$
- **Overflow** occurs if:
 - $A \geq 0$ y $B \geq 0$ and $A + B < 0$
 - $A < 0$ y $B < 0$ and $A + B \geq 0$
- **Example:** $A=0111$ and $B=0101$: $-A=1001$ and $-B=1011$
 - $A + B = 0111 + 0101 = 1100$ y $C_f = 0$: overflow
 - $A - B = A + (-B) = 0111 + 1011 = 0010$ y $C_f = 1$
 - $-A + B = 1001 + 0101 = 1110$ y $C_f = 0$
 - $-A - B = (-A) + (-B) = 1001 + 1011 = 0100$ y $C_f = 1$: overflow



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Fixed point representations

BCD: Binary Coded Decimal

- Used to represent decimal digits in binary;
- Four bits represent one decimal digit:

Decimal digit	BCD	Decimal digit	BCD
0	0000	5	0101
1	0001	6	0110
2	0010	7	0111
3	0011	8	1000
4	0100	9	1001

- To represent decimal numbers with more digits just group BCD packages
- Example: 73 → 0111 0011



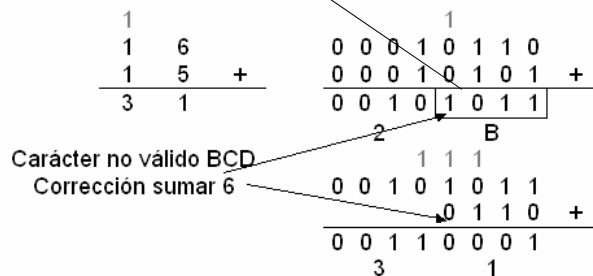
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Addition in BCD

Valores válidos BCD	Valores NO válidos BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111



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Addition in hexadecimal

+	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	0
2	2	3	4	5	6	7	8	9	A	B	C	D	E	F	0	1
3	3	4	5	6	7	8	9	A	B	C	D	E	F	0	1	2
4	4	5	6	7	8	9	A	B	C	D	E	F	0	1	2	3
5	5	6	7	8	9	A	B	C	D	E	F	0	1	2	3	4
6	6	7	8	9	A	B	C	D	E	F	0	1	2	3	4	5
7	7	8	9	A	B	C	D	E	F	0	1	2	3	4	5	6
8	8	9	A	B	C	D	E	F	0	1	2	3	4	5	6	7
9	9	A	B	C	D	E	F	0	1	2	3	4	5	6	7	8
A	A	B	C	D	E	F	0	1	2	3	4	5	6	7	8	9
B	B	C	D	E	F	0	1	2	3	4	5	6	7	8	9	A
C	C	D	E	F	0	1	2	3	4	5	6	7	8	9	A	B
D	D	E	F	0	1	2	3	4	5	6	7	8	9	A	B	C
E	E	F	0	1	2	3	4	5	6	7	8	9	A	B	C	D
F	F	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E

Los valores implican que me llevo 1 de acarreo

$$\begin{array}{r}
 1 \\
 \text{C} \quad \text{A} \quad \text{F} \quad \text{E} \\
 \text{1} \quad \text{2} \quad \text{F} \quad \text{4} \quad + \\
 \hline
 \text{D} \quad \text{C} \quad \text{F} \quad \text{2}
 \end{array}$$

$$\begin{array}{r}
 1 \quad 1 \quad 1 \quad 1 \\
 \text{F} \quad \text{A} \quad \text{B} \quad \text{E} \\
 \text{C} \quad \text{A} \quad \text{F} \quad \text{E} \quad + \\
 \hline
 \text{1} \quad \text{C} \quad \text{5} \quad \text{B} \quad \text{C}
 \end{array}$$



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Alphanumeric representations (I)

- Represent each character (7, A, j, =, *,) by a group of bits.
- Examples
 - 6 bits ($2^6=64$ characters): Fielddata and BCDIC
 - 7 bits ($2^7=128$ characters): ASCII
 - 8 bits ($2^8=256$ characters): extended ASCII and EBCDIC
 - 16 bits ($2^{16}=65536$ characters): UNICODE



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Alphanumeric representations (II)

- Phrases are represented grouping characters. Options:

- Fixed length string

P	E	P	E	A	N	T	O	N	I	O	R	O	S	A						
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--	--	--	--	--	--

- Variable length string

- Delimiter character

*	P	E	P	E	*	A	N	T	O	N	I	O	*	R	O	S	A
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

- Explicit length

4	P	E	P	E	7	A	N	T	O	N	I	O	4	R	O	S	A
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---



Alphanumeric representations (III) ASCII code

850 Multilingual (Latin 1)	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255																																
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255

