



# Unit 4.2

## Sequential Systems Design



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### Bibliography

- Digital Design.  
M. Morris Mano. Prentice-Hall
- Introduction to Digital Logic Design.  
John P. Hayes. Addison-Wesley

# Introduction



- Systematic way of designing any machine that passes through different states.

Examples: Counter, traffic lights, vending machine...

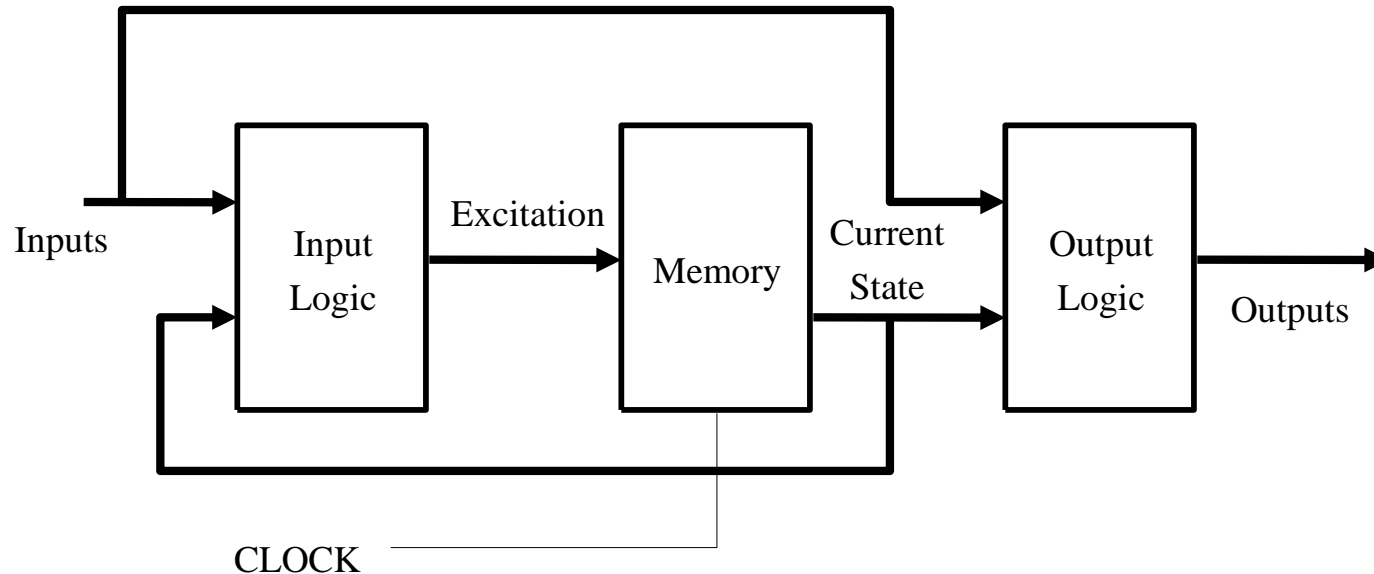
- Generally called ***Finite State Machines/Automatas***

- Two types:

**Mealy** Machines

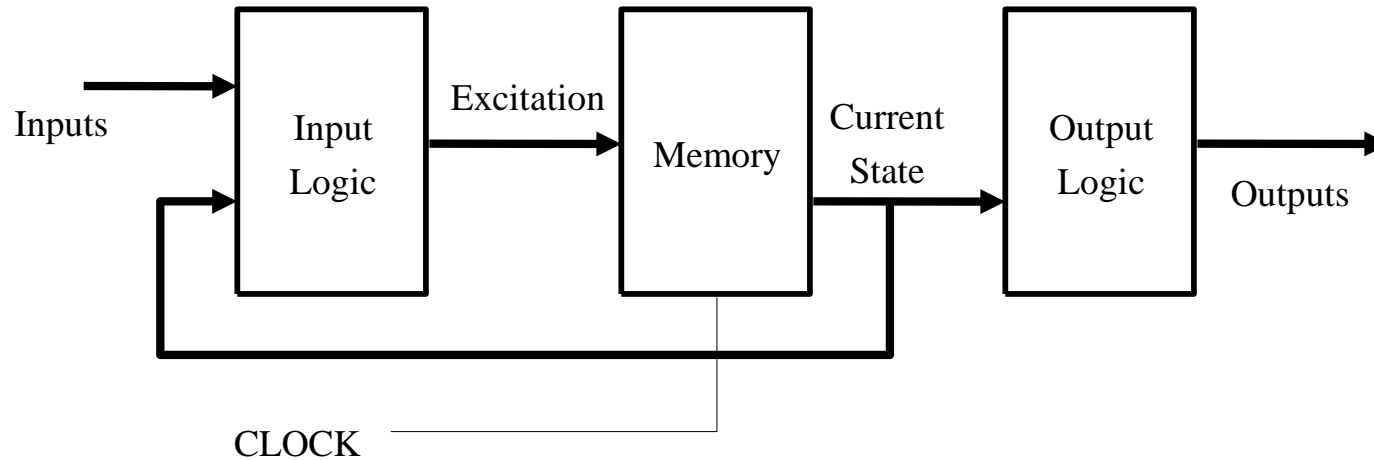
**Moore** Machines

# Mealy Machine



**Outputs** are a function of both **inputs** and **current state**

# Moore Machine



**Outputs are a function of current state only**

# Steps to design

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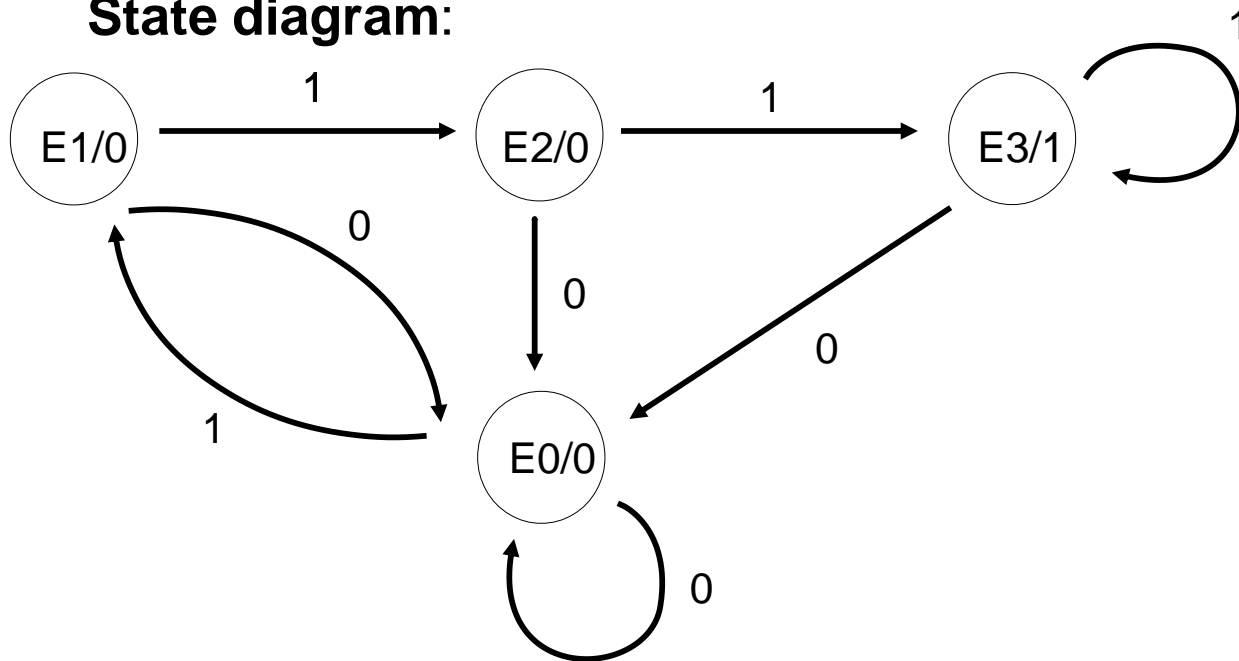


- 1- Understand specifications of the problem
- 2- Choose Mealy/Moore based on simplicity
- 3- Draw state diagram
- 4- Codify states and outputs and choose flip-flops
- 5- Obtain output function
- 6- Write transition and excitation table
- 7- Obtain and simplify excitation functions
- 8- Implement the circuit

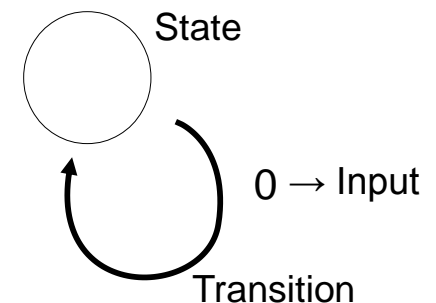
# Example 1: Moore (I)

Design a Moore automata that detects a sequence of tree or more “1” in the input: ...111...

**State diagram:**



E0/0 → State Name/Output



# Example 1: Moore (II)

## Codify states and outputs:

- There are 4 states so we need two bits to codify them
- We use two JK flip-flops
- Codification:

States	JKs		Output
	Q1	Q0	Z
E0	0	0	0
E1	0	1	0
E2	1	0	0
E3	1	1	1

## Obtain output function:

$$Z = Q1 Q0$$



# Example 1: Moore (III)

Write transition and excitation table:

Current state	Input	Next state	JK excitation			
Q1 <sup>t</sup> Q0 <sup>t</sup>	Y	Q1 <sup>t+1</sup> Q0 <sup>t+1</sup>	J1	K1	J0	K0
E0: 0 0	0	0 0	0	X	0	X
E0: 0 0	1	0 1	0	X	1	X
E1: 0 1	0	0 0	0	X	X	1
E1: 0 1	1	1 0	1	X	X	1
E2: 1 0	0	0 0	X	1	0	X
E2: 1 0	1	1 1	X	0	1	X
E3: 1 1	0	0 0	X	1	X	1
E3: 1 1	1	1 1	X	0	X	0

JK Excitation table

Q <sup>t</sup>	Q <sup>t+1</sup>	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0



# Example 1: Moore (IV)

**Obtain and simplify excitation function:**

-Obtain  $J_1$ ,  $K_1$ ,  $J_0$  and  $K_0$  in terms of  $Q_1^{t+1}$ ,  $Q_0^{t+1}$  and  $Y$  using Karnaugh

-Example

$$J_1 = Q_0 Y$$

$Q_1 \setminus Q_0 Y$	00	01	11	10
0			1	
1	X	X	X	X

Doing the other Karnaugh Maps:

$$K_1 = Y!$$

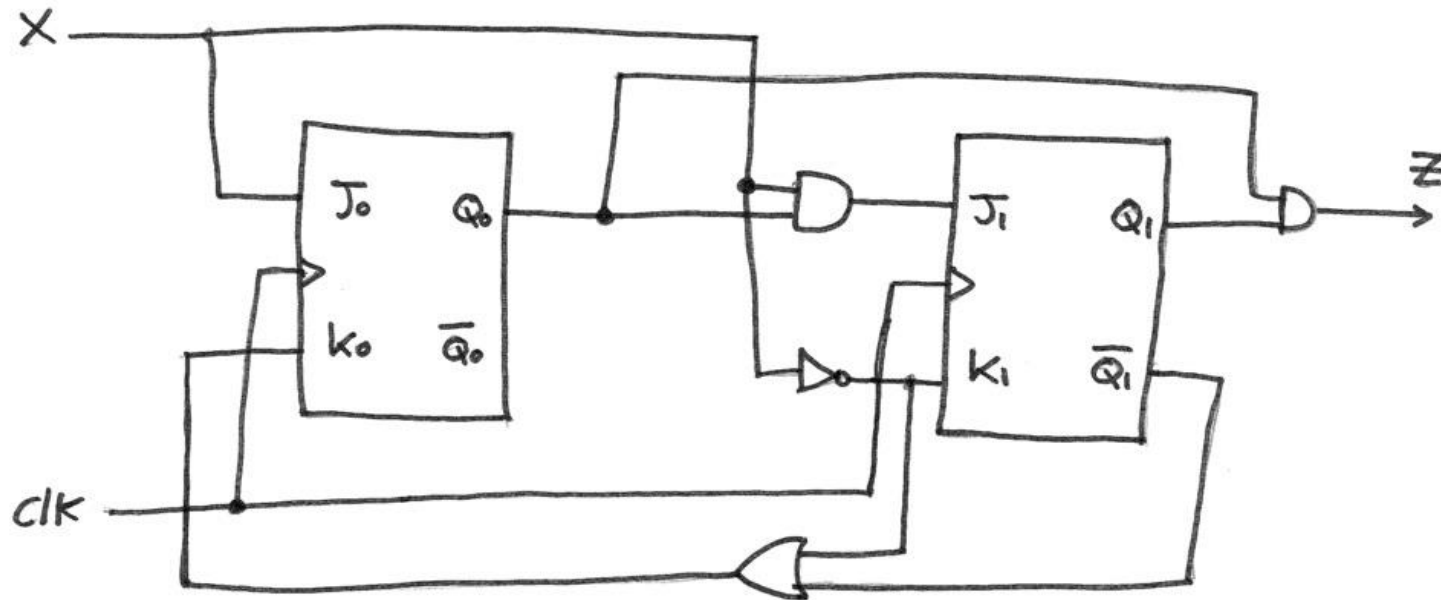
$$J_0 = Y$$

$$K_0 = Q_1! + Y!$$

# Example 1: Moore (V)



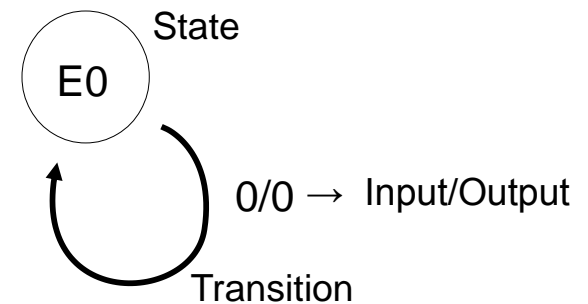
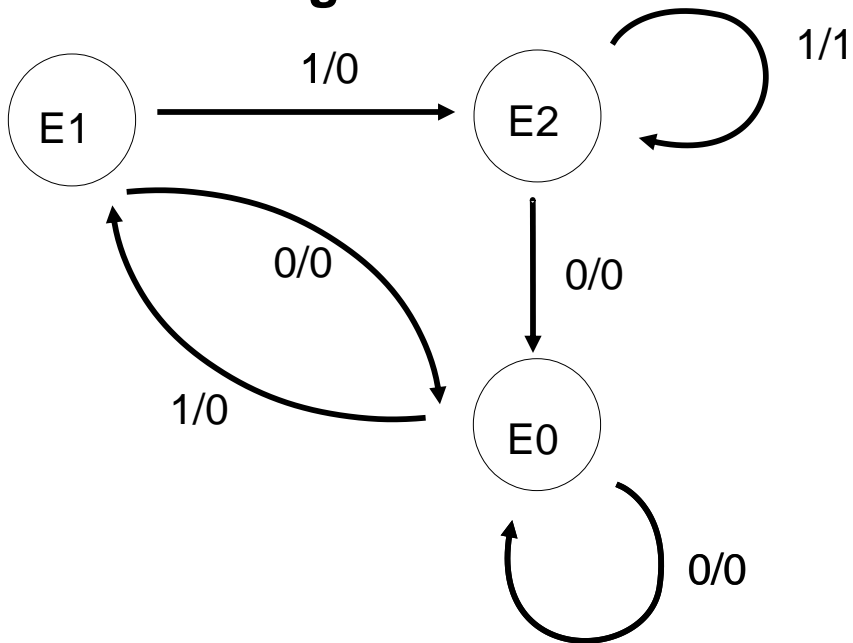
Implement the circuit:



## Example 2: Mealy (I)

Design a Mealy automata that detects a sequence of three or more “1” in the input: ...111...

**State diagram:**



## Example 2: Mealy (II)

### Codify states and outputs:

- There are 3 states so we need two bits to codify them
- We use two JK flip-flops
- Codification:

### Obtain output function:

Q1 \ Q0Y	00	01	11	10
0				
1		1	X	X

$$Z = Y Q1$$

States	JKs		Input	Output
	Q1	Q0	Y	Z
E0	0	0	0	0
E0	0	0	1	0
E1	0	1	0	0
E1	0	1	1	0
E2	1	0	0	0
E2	1	0	1	1
E3	1	1	0	X
E3	1	1	1	X



## Example 2: Mealy (III)

Write transition and excitation table:

Current state	Input	Next state	JK excitation			
Q1 <sup>t</sup> Q0 <sup>t</sup>	Y	Q1 <sup>t+1</sup> Q0 <sup>t+1</sup>	J1	K1	J0	K0
E0: 0 0	0	0 0	0	X	0	X
E0: 0 0	1	0 1	0	X	1	X
E1: 0 1	0	0 0	0	X	X	1
E1: 0 1	1	1 0	1	X	X	1
E2: 1 0	0	0 0	X	1	0	X
E2: 1 0	1	1 0	X	0	0	X
E3: 1 1	0	X X	X	X	X	X
E3: 1 1	1	X X	X	X	X	X

JK Excitation table

Q <sup>t</sup>	Q <sup>t+1</sup>	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

## Example 2: Mealy (IV)

**Obtain and simplify excitation function:**

-Obtain  $J_1$ ,  $K_1$ ,  $J_0$  and  $K_0$  in terms of  $Q_1^{t+1}$ ,  $Q_0^{t+1}$  and  $Y$  using Karnaugh

-Example

$$J_1 = Q_0 Y$$

$Q_1 \setminus Q_0 Y$	00	01	11	10
0			1	
1	X	X	X	X

Doing the other Karnaugh Maps:

$$K_1 = Y!$$

$$J_0 = Y Q_1!$$

$$K_0 = 1$$

# Example 2: Mealy (V)



Implement the circuit:

